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Quadrupole moments of medium-weight and heavy hypernuclei with the NΛ configurations (N = p and n)

Y. Tanaka[1,a] and M. Matoba[2]

[1] Department of Physics, Kyushu Kyoritsu University, Kita-Kyushu 807, Japan
[2] Department of Nuclear Engineering, Kyushu University, Fukuoka 812, Japan

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Abstract. Using the shell model wave functions, we have studied quadrupole moments of medium-weight and heavy hypernuclei, and obtained the shell model values of quadrupole moments of NΛ systems (N = p and n). With the use of the first-order perturbation theory, we have also estimated the configuration mixing effects on quadrupole moments of these NΛ hypernuclei. We show that the hyperon-induced configuration mixing effects are small and the nucleon-induced configuration mixing effects are large in many cases.

PACS. 21.80.+a Hypernuclei

1 Introduction

Motoba *et al.* have investigated $E2$ transition moments of light Λ-hypernuclei by using the cluster model [1,2]. They showed the several times enhancement of $B(E2)$ values in comparison with the simple shell model values, and pointed out the importance of the clustering effects in hypernuclei. Recently, we have calculated quadrupole moments of light Λ-hypernuclei on the basis of $N = Z$ double-closed core, and estimated the configuration mixing effects on these hypernuclear quadrupole moments [3]. To study electromagnetic moments of hypernuclei will serve to understand the hypernuclear structure and to refine the hypernuclear model.

In this paper, within the framework of shell model, on the basis of $N \neq Z$ double-closed nuclear core, we calculate quadrupole moments of medium-weight and heavy hypernuclei with the NΛ configuration (N = p and n), and estimate the configuration mixing effects on these hypernuclear quadrupole moments by using the first-order perturbation theory [4,5]. To investigate hypernuclear quadrupole moments will lead us to understand the quadrupole deformation of hypernuclei and the effective charges of hyperons as well as nucleons in hypernuclei.

In section 2, quadrupole moments of Λ-hypernuclei are discussed on the basis of the perturbation theory. In section 3, are presented the results of shell model calculations. In section 4, we discuss quadrupole moments of some special hypernuclei and effective charges of nucleons and hyperons in hypernuclei

In section 5, a summary is given. In appendix, are shown the correction formulae of configuration mixing effects on static moments of medium-weight and heavy NΛ hypernuclei.

[a] e-mail: ytanaka@kyukyo-u.ac.jp

2 Static moments of Λ-hypernuclei

Previously, within the framework of shell model, we have studied magnetic moments of Λ-hypernuclei and estimated the configuration mixing effects on these hypernuclei by using the first-order perturbation theory [6,7]. Recently, to investigate quadrupole moments of light Λ-hypernuclei [3], we have used the same method as previous works of hypernuclear magnetic moments. In this paper, we study quadrupole moments of medium-weight and heavy Λ-hypernuclei with the use of the same method as previous works [3,6,7].

Here, we review and summarize the first-order perturbation method to investigate static moments of Λ-hypernuclei within the framework of the shell model. The wave functions of Λ-hypernuclei may be written as

$$\Psi(J) = \Psi_0 + \sum_i \gamma_{\mathrm{N}i} \Psi_{\mathrm{N}i} + \sum_i \gamma_{\Lambda i} \Psi_{\Lambda i}, \qquad (1)$$

if the admixture of the excited to the ground configuration is very small ($\gamma_{\mathrm{N}i} \ll 1$ and $\gamma_{\Lambda i} \ll 1$). The zeroth-order wave function Ψ_0 is given by a nuclear wave function (Ψ_N) and a Λ-hyperon wave function (Ψ_Λ); $\Psi_0 = [\Psi_\mathrm{N} \times \Psi_\Lambda]_0$. The basic wave functions $\Psi_{\mathrm{N}i}$ and $\Psi_{\Lambda i}$ are constructed by exciting a nucleon (N) and a hyperon (Λ), respectively; $\Psi_{\mathrm{N}i} = [\Psi_\mathrm{N}{}' \times \Psi_\Lambda]_i$ and $\Psi_{\Lambda i} = [\Psi_\mathrm{N} \times \Psi_\Lambda{}']_i$. The coefficients $\gamma_{\mathrm{N}i}$ and $\gamma_{\Lambda i}$ are the mixing amplitudes, which may be evaluated by the perturbation theory. They are written as

$$\gamma_{Ni} = \frac{\langle \Psi_{Ni} | \sum V | \Psi_0 \rangle}{E_0 - E_{Ni}}, \quad (2)$$

$$\gamma_{\Lambda i} = \frac{\langle \Psi_{\Lambda i} | \sum V | \Psi_0 \rangle}{E_0 - E_\Lambda}, \quad (3)$$

$$\sum V = \sum V_{NN} + \sum V_{N\Lambda}, \quad (4)$$

where V_{NN} and $V_{N\Lambda}$ denote the residual nucleon-nucleon interaction and nucleon-hyperon interaction, respectively. The nucleon-induced corrections, which are induced by the nucleon-nucleon interaction (V_{NN}), were extensively studied by Blin-Stoyle, Arima and Horie [4,5]. On the other hand, the hyperon-induced corrections, which are induced by the nucleon-hyperon interaction ($V_{N\Lambda}$), are attempted to estimate in this and the previous papers [3,6,7].

The hypernuclear spin (J) is obtained by coupling a nuclear spin (J_N) and a Λ-hyperon spin (J_Λ); $\tilde{J} = \tilde{J}_N + \tilde{J}_\Lambda$. With the use of the wave function (1), we can calculate hypernuclear moment

$$f^{(k)} = f_N^{(k)} + f_\Lambda^{(k)} \quad (\mathrm{N} = \mathrm{p\ and\ n}), \quad (5)$$

where the symbol $f^{(k)}$ denotes the one-body operator of rank k, such as the magnetic moment $\mu(k=1)$ and the quadrupole moment $\mathcal{Q}(k=2)$. The result is written in the form

$$\langle f^{(k)}(J) \rangle = \langle f^{(k)}(J) \rangle_0 + \delta_N + \delta_\Lambda,$$

$$\langle f^{(k)}(J) \rangle = \langle \Psi_0 | f^{(k)} | \Psi_0 \rangle_J + \sum_i 2\gamma_{Ni} \langle \Psi_0 | f^{(k)} | \Psi_{Ni} \rangle_J + \sum_i 2\gamma_{\Lambda i} \langle \Psi_0 | f^{(k)} | \Psi_{\Lambda i} \rangle_J. \quad (6)$$

Here, $\langle f^{(k)}(J) \rangle_0$ shows the zeroth-order value of hypernuclear static moments, and δ_N and δ_Λ denote the first-order corrections induced by nucleon (N) excitations and hyperon (Λ) excitations, respectively.

After some Racah algebra [8], we have obtained the formulae of configuration mixing effects on static moments of NΛ hypernuclei. The results are summarized in appendix.

3 Shell model calculations

Within the framework of shell model, we have estimated the first-order correction δ_N to quadrupole moments of medium-weight and heavy hypernuclei with the simple NΛ configurations (N = p and n). The correction δ_Λ turns out to be zero, as far as we use the free charge of a Λ-hyperon ($e_\Lambda = 0$). The excited states ($\Psi_{N1}, \Psi_{N2}, \Psi_{N3}, \dots$) are classified according to the configuration mixing theory of Arima and Horie [5]. Employing the one-proton excitation mode of Arima and Horie [5], we have obtained the correction formulae with the use of the standard Racah algebra [8]. The correction δ_N is divided into two parts:

$$\delta_N = \delta_{NN} + \delta_{N\Lambda},$$

$$\delta_N = \sum_i \delta_{pi}(\mathrm{NN}) + \sum_i \delta_{pi}(\mathrm{N}\Lambda) \quad (i = 1, 2, 3, \dots), \quad (7)$$

where δ_{NN} and $\delta_{N\Lambda}$ denote the corrections induced by the NN interactions (V_{NN}) and the NΛ interactions ($V_{N\Lambda}$), respectively. The symbol δ_{pi}(NN) shows the correction of the one-proton excitation mode induced by the NN interaction (V_{NN}), which was extensively studied in the nuclear moment analyses by Arima and Horie [5]. On the other hand, $\delta_{pi}(\mathrm{N}\Lambda)$ denotes the correction of the one-proton excitation mode induced by the NΛ interactions ($V_{N\Lambda}$), which we attempt to estimate in the hypernuclear moment analyses [3,6,7].

3.1 The pΛ systems with $j_p = l_p \pm 1/2$ orbit

We use the following basic states:

$$\Psi_0 = |j_p j_\Lambda\rangle_J, \quad (8)$$

$$\Psi_{N1} = |j_p{}' j_\Lambda\rangle_J, \quad (9)$$

$$\Psi_{N2} = a|j_p{}' j_\Lambda\rangle_J + b|[[j_p j_n{}^{-1}](J_0 = 0) j_n{}'](J_N) j_\Lambda\rangle_J, \quad (10)$$

$$\Psi_{N3} = |[(j_p{}')^{-1} j_p{}^2 (J_0)](J_N) j_\Lambda\rangle_J, \quad (11)$$

$$\Psi_{N4} = |[[(j_\pi{}^{-1} j_\pi{}'](J_0) j_p](J_N) j_\Lambda\rangle_J, \quad (12)$$

$$\Psi_{N5} = \alpha|[[(j_\pi{}^{-1} j_\pi{}'](J_0) j_p](J_N) j_\Lambda\rangle_J + \beta|[[(j_\nu{}^{-1} j_\nu{}'](J_0) j_p](J_N) j_\Lambda\rangle_J + \{\text{2h-4p states}\}. \quad (13)$$

These states are schematically shown in fig. 1 and fig. 2. The factors (a, b) and $(\alpha, \beta, \gamma, \delta)$ in these wave functions (Ψ_{Ni}) are given by the isospin Clebsch-Gordan coefficients as follows:

$$(a, b) = \left(\sqrt{\frac{2T_c}{2T_c + 1}}, -\sqrt{\frac{1}{2T_c + 1}} \right), \quad (14)$$

$$(\alpha, \beta, \gamma, \delta) = \left(\sqrt{\frac{T_c}{2(T_c + 1)}}, \sqrt{\frac{T_c}{2(T_c + 1)}}, -\sqrt{\frac{1}{2(T_c + 1)}}, -\sqrt{\frac{1}{2(T_c + 1)}} \right), \quad (15)$$

where the quantum number T_c denotes the isospin of the nuclear core. On the other hand, the symbol T_0 shows the isospin of valence particles. Hypernuclear excited states (Ψ_{Ni}) are obtained according to the Arima-Horie classifications scheme of the configuration mixing theory [5]. For example, in the case of ${}^{50}_{\Lambda}$Sc, we get the wave functions Ψ_{N2} and Ψ_{N5} by an application of the isospin lowering operator (T_-) to $|{}^{50}_{\Lambda}\mathrm{Ca}(\text{gr. conf.})\rangle$ and $|{}^{50}_{\Lambda}\mathrm{Ca}(\text{ex. conf.})\rangle$, respectively. The total isospin of Ψ_{N5} are obtained by the core-particles coupling scheme: $T = T_c \times T_0(\mathrm{hp})$. In the numerical calculations, neglecting {2h-4p states}, we select only the one-proton excitation mode, because we are interested in the first-order corrections to hypernuclear quadrupole moments.

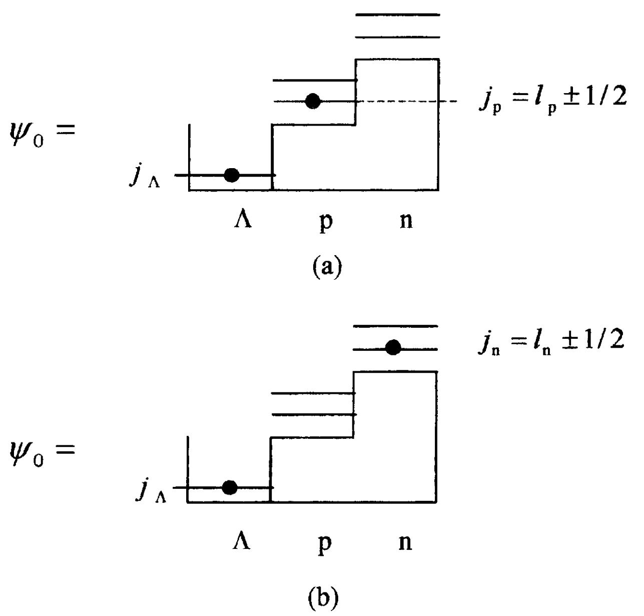

Fig. 1. The shell model configuration of the ground states of medium-weight and heavy hypernuclei. (a) pΛ systems with $j_\mathrm{p} = l_\mathrm{p} \pm 1/2$ orbit. (b) nΛ systems with $j_\mathrm{n} = l_\mathrm{n} \pm 1/2$ orbit.

3.2 The nΛ systems with $\mathrm{j_n = l_n \pm 1/2}$ orbit.

Basic states are as follows:

$$\Psi_0 = |j_\mathrm{n} j_\Lambda\rangle_J, \tag{16}$$

$$\Psi_{\mathrm{N}1} = |[j_\pi{}^{-1}[j_\pi{}' j_\mathrm{n}](J_0)](J_\mathrm{N}) j_\Lambda\rangle_J, \tag{17}$$

$$\begin{aligned}\Psi_{\mathrm{N}2} = &\tilde{\alpha}|[(j_\mathrm{p}{}')^{-1}[j_\mathrm{p} j_\mathrm{n}](J_0, T_0 = 1)(J_\mathrm{N}) j_\Lambda\rangle_J \\ &+\delta|[(j_\mathrm{n}{}')^{-1} j_\mathrm{n}^2 (J_0, T_0 = 1)](J_\mathrm{N}) j_\Lambda\rangle_J \\ &+ \{\text{4p-2h states}\},\end{aligned} \tag{18}$$

$$\Psi_{\mathrm{N}3} = |[(j_\mathrm{p}{}')^{-1}[j_\mathrm{p} j_\mathrm{n}](J_0, T_0 = 0)(J_\mathrm{N}) j_\Lambda\rangle_J, \tag{19}$$

$$\begin{aligned}\Psi_{\mathrm{N}4} = &\alpha|[j_\pi{}^{-1}[j_\pi{}' j_\mathrm{n}](J_0, T_0 = 1)](J_\mathrm{N}) j_\Lambda\rangle_J \\ &+ \beta|[j_\pi{}^{-1}[j_\mathrm{p} j_\nu{}'](J_0, T_0 = 1)](J_\mathrm{N}) j_\Lambda\rangle_J \\ &+ \delta|[j_\nu{}^{-1}[j_\mathrm{n} j_\nu{}'](J_0, T_0 = 1)](J_\mathrm{N}) j_\Lambda\rangle_J \\ &+ \{\text{4p-2h states}\},\end{aligned} \tag{20}$$

$$\begin{aligned}\Psi_{\mathrm{N}5} = &\frac{1}{\sqrt{2}}|[j_\pi{}^{-1}[j_\pi{}' j_\mathrm{n}](J_0, T_0 = 0)](J_\mathrm{N}) j_\Lambda\rangle_J \\ &+ \frac{1}{\sqrt{2}}|[j_\pi{}^{-1}[j_\mathrm{p} j_\nu{}'](J_0, T_0 = 0)](J_\mathrm{N}) j_\Lambda\rangle_J.\end{aligned} \tag{21}$$

These wave functions are schematically presented in fig. 1 and fig. 3. The factors $(\alpha, \beta, \gamma, \delta)$ in these states are given by eq. (15) ($\tilde{\alpha} = \sqrt{2}\alpha$). In the case of ${}^{50}_{\Lambda}$Ca, for example, we obtain the wave functions $\Psi_{\mathrm{N}2}$ and $\Psi_{\mathrm{N}4}$ by applying the isospin shift operator (T_-) to $|{}^{50}_{\Lambda}\mathrm{K}(\text{gr. conf.})\rangle$ and $|{}^{50}_{\Lambda}\mathrm{K}(\text{ex. conf.})\rangle$, respectively. The total isospin of $\Psi_{\mathrm{N}2}$ and $\Psi_{\mathrm{N}4}$ are obtained by the equation: $T = T_\mathrm{c} \times T_0(\mathrm{pp})$. In the numerical studies, as for the pΛ systems, we choose only the one-proton excitation mode, because we try to estimate the first-order corrections to hypernuclear quadrupole moments.

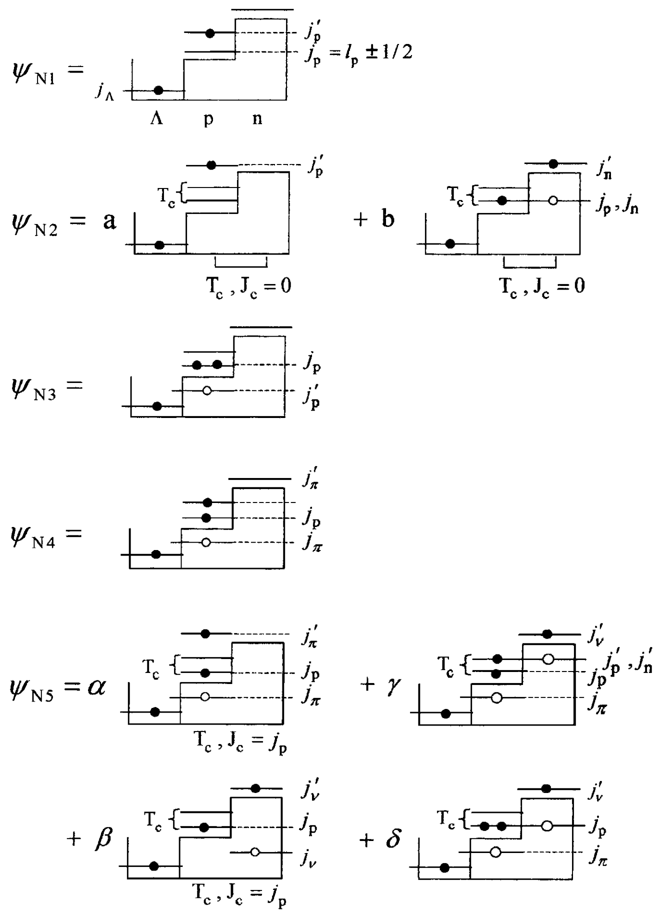

Fig. 2. The shell model configuration of the excited states of medium-weight and heavy pΛ hypernuclei with $j_\mathrm{p} = l_\mathrm{p} \pm 1/2$ orbit.

Hypernuclear wave functions with good isospin, such as $\Psi_{\mathrm{N}2}$, $\Psi_{\mathrm{N}5}$ (in fig. 2), $\Psi_{\mathrm{N}2}$ and $\Psi_{\mathrm{N}4}$ (in fig. 3) were used to investigate magnetic moments of medium-weight and heavy hypernuclei [7]. On the other hand, nuclear wave functions with good isospin were proposed for a systematic study of core polarization phenomena of medium-weight and heavy nuclei, such as $N = 28$ isotones [9].

The correction formulae for static moments of NΛ hypernuclei are presented in appendix, where the symbol $f_\mathrm{p}^{(k)}$ denotes the one-body operator of rank k, such as the quadrupole moment operator $\mathcal{Q}_\mathrm{p} = \sqrt{16\pi/5}\, e_\mathrm{p} r_\mathrm{p}^2 Y_{20}(\theta_\mathrm{p}, \phi_\mathrm{p})$, and the symbol $\tilde{\delta}_{\mathrm{p}i}$ shows the reduced matrix element of the correction

$$\delta_{\mathrm{p}i} = \begin{pmatrix} J & k & J \\ -J & 0 & J \end{pmatrix} \tilde{\delta}_{\mathrm{p}i} \quad (i = 1, 2, 3, \dots). \tag{22}$$

In the shell model analyses, assuming the energy denominator $-\Delta E_{\mathrm{p}i} = E_0 - E_{\mathrm{p}i}$ is constant, we summed up the intermediate states and described the corrections $\tilde{\delta}_{\mathrm{p}i}$ in terms of the average energies $\bar{E}_{\mathrm{p}i}(\mathrm{NN}, j_\pi j_\pi{}')$, $\bar{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')$ and $\tilde{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')$, which are defined in appendix.

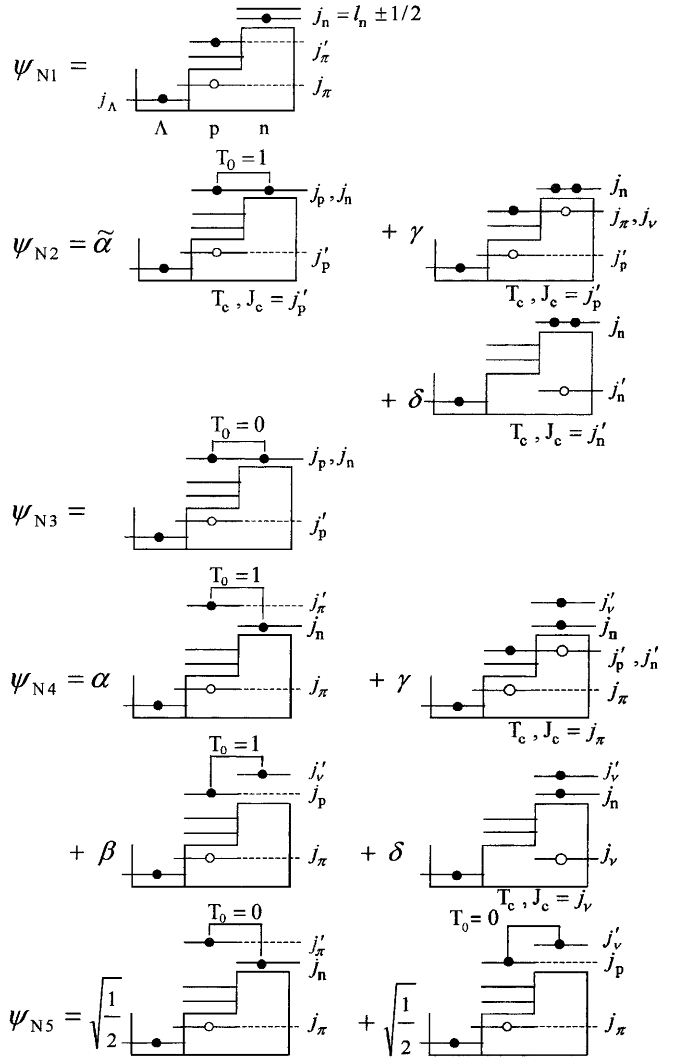

Fig. 3. The shell model configuration of the excited states of medium-weight and heavy nΛ hypernuclei with $j_{\rm n} = l_{\rm n} \pm 1/2$ orbit.

3.3 Numerical results

The zeroth-order value of quadrupole moments of NΛ hypernuclei $j_{\rm N} = l_{\rm N} \pm 1/2$ is given by the equation [3]

$$\langle \mathcal{Q}(J)\rangle_0 = \begin{pmatrix} J & 2 & J \\ -J & 0 & J \end{pmatrix} \langle j_{\rm N} j_\Lambda J \| \mathcal{Q}_{\rm N} \| j_{\rm N} j_\Lambda J\rangle,$$
$$\langle \mathcal{Q}(J)\rangle_0 = \alpha(J)\langle \mathcal{Q}_{\rm N}(j_{\rm N})\rangle, \qquad (23)$$

where $\langle \mathcal{Q}_{\rm N}(j_{\rm N})\rangle$ shows the single-particle value of the nuclear quadrupole moment and the coefficient $\alpha(J)$ is given as follows:

$$\alpha(J) = \begin{cases} \dfrac{(2j_{\rm N}-2)(2j_{\rm N}+3)}{2j_{\rm N}(2j_{\rm N}+1)}, & J = j_{\rm N} - 1/2, \\ 1, & J = j_{\rm N} + 1/2. \end{cases} \qquad (24)$$

Numerical values of pΛ and nΛ hypernuclear quadrupole moments are listed in table 1 and table 2, respectively. Effective values are calculated by using observed values of quadrupole moments of nuclei with {core + N} configurations (N = p and n), such as ^{209}Bi and ^{73}Ge [10]. On the other hand, single-particle values are estimated simply assuming the uniform charge distribution with a nuclear radius $R = r_0 A^{1/3}$ [3].

It is interesting to discuss effective quadrupole moments of heavier NΛ hypernuclei, such as $^{210}_{\Lambda}$Bi and $^{74}_{\Lambda}$Ge. The discussions on these moments are given in the next section.

Here we show and discuss the configuration mixing effects on quadrupole moments of medium-weight and heavy NΛ hypernuclei. The calculations of these heavier hypernuclei are rather tedious, and we select the simple cases; $^{50}_{\Lambda}$Sc and $^{50}_{\Lambda}$Ca, the pΛ system and the nΛ system, respectively. With the use of the correction formulae in appendix, we can estimate the configuration mixing effects on quadrupole moments of these hypernuclei.

The active orbits in the calculations are as follows:

$[^{50}_{\Lambda}{\rm Sc}]\Psi_{\rm p2} : j_{\rm p} = 1f_{7/2};$
$j_{\rm p}{}' = 1f_{5/2}, 2p_{3/2}, 1h_{11/2}, 1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}.$
$\Psi_{\rm p3} : j_{\rm p} = 1f_{7/2}; j_{\rm p}{}' = 1p_{3/2}.$
$\Psi_{\rm p5} : j_\pi = 1d_{5/2};$
$j_\pi{}' = 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 1d_{3/2}; j_\pi{}' = 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 2s_{1/2}; j_\pi{}' = 2d_{5/2}, 2d_{3/2}.$
$j_\pi = 1p_{3/2}; j_\pi{}' = 1f_{5/2}, 2p_{3/2}, 2p_{1/2}.$
$j_\pi = 1p_{1/2}; j_\pi{}' = 1f_{5/2}, 2p_{3/2}.$

$[^{50}_{\Lambda}{\rm Ca}]\Psi_{\rm p1} : j_\pi = 1p_{3/2}; j_\pi{}' = 1f_{7/2}.$
$\Psi_{\rm p2} : j_{\rm p} = 2p_{3/2}; j_{\rm p}{}' = 1p_{3/2}, 1p_{1/2}.$
$\Psi_{\rm p3} : j_{\rm p} = 2p_{3/2}; j_{\rm p}{}' = 1p_{3/2}, 1p_{1/2}.$
$\Psi_{\rm p4} : j_\pi = 1d_{5/2};$
$j_\pi{}' = 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 1d_{3/2}; j_\pi{}' = 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 2s_{1/2}; j_\pi{}' = 2d_{5/2}, 2d_{3/2}.$
$j_\pi = 1p_{3/2}; j_\pi{}' = 1f_{5/2}, 2p_{1/2}.$
$j_\pi = 1p_{1/2}; j_\pi{}' = 1f_{5/2}.$
$\Psi_{\rm p5} : j_\pi = 1d_{5/2};$
$j_\pi{}' = 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 1d_{3/2}; j_\pi{}' = 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}.$
$j_\pi = 2s_{1/2}; j_\pi{}' = 2d_{5/2}, 2d_{3/2}.$
$j_\pi = 1p_{3/2}; j_\pi{}' = 1f_{5/2}, 2p_{1/2}.$
$j_\pi = 1p_{1/2}; j_\pi{}' = 1f_{5/2}.$

All configurations ($\Psi_{\rm p1}$–$\Psi_{\rm p5}$) are not always used to evaluate the first-order corrections to hypernuclear quadrupole moments. A Λ-hyperon is assumed to be in the ground configuration; $j_\Lambda = 1s_{1/2}$. In tables 3 and 4, the numerical results of configuration mixing effects are given in unit of the inverse of the harmonic oscillator frequency $1/\nu_{\rm N}$

Table 1. Quadrupole moments of medium-weight and heavy pΛ hypernuclei with $j_\mathrm{p} = l_\mathrm{p} \pm 1/2$ orbit (in efm^2).

hypernuclei	Configuration	J^π	Effective[(a)]	Single particle[(b)]
${}^{38}_{\Lambda}$Cl	$1d_{3/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	1^+	-3.246	-1.918
		2^+	-6.493	-3.837
${}^{50}_{\Lambda}$Sc	$1f_{7/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	3^-	*	-6.886
		4^-	*	-7.712
${}^{90}_{\Lambda}$Y	$1g_{9/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	4^+	*	-11.690
		5^+	*	-12.525
${}^{92}_{\Lambda}$Nb	$1g_{9/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	4^+	*	-11.865
		5^+	*	-12.712
${}^{134}_{\Lambda}$Sb	$1h_{11/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	5^-	*	-16.529
		6^-	*	-17.316
${}^{210}_{\Lambda}$Bi	$1i_{13/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	6^+	*	-23.540
		7^+	*	-24.342
	$1h_{9/2}$ (p) $\times$ $1s_{1/2}(\Lambda)$	4^-	-42.9	-20.654
		5^-	-46.0	-22.129

(a) Effective Values are calculated by observed values of quadrupole moments of nuclei with the {core+p} configuration.
(b) Single-particle values are calculated by assuming the uniform charge distribution with a nuclear radius $R = r_0 A^{1/3}$.

Table 2. Quadrupole moments of medium-weight and heavy nΛ hypernuclei with $j_\mathrm{n} = l_\mathrm{n} \pm 1/2$ orbit (in efm^2).

Hypernuclei	Configuration	J^π	Effective[(a)]	Single particle[(b)]
${}^{50}_{\Lambda}$Ca	$2p_{3/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	1^-	*	0.0
		2^-	*	0.0
${}^{62}_{\Lambda}$Ni	$1f_{5/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	2^-	-6.4	0.0
		3^-	-8.0	0.0
${}^{74}_{\Lambda}$Ge	$1g_{9/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	4^+	-16.1	0.0
		5^+	-17.3	0.0
${}^{90}_{\Lambda}$Sr	$2d_{5/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	2^+	*	0.0
		3^+	*	0.0
${}^{92}_{\Lambda}$Zr	$2d_{5/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	2^+	*	0.0
		3^+	*	0.0
${}^{134}_{\Lambda}$Sn	$2f_{7/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	3^-	*	0.0
		4^-	*	0.0
${}^{210}_{\Lambda}$Pb	$2g_{9/2}$(n) $\times$ $1s_{1/2}(\Lambda)$	4^+	*	0.0
		5^+	*	0.0

(a) Effective values are calculated by observed values of quadrupole moments of nuclei with the core {core+n} configuration.
(b) Same comments as for table 1

and $1/\nu_\Lambda$. The factors $\bar{\varepsilon}_{\mathrm{p}i}(\mathrm{NN}, j_\pi j_\pi{}')$ and $\bar{\varepsilon}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')$ in these tables are defined as follows:

$$\bar{\varepsilon}_{\mathrm{p}i}(\mathrm{NN}, j_\pi j_\pi{}') = \frac{\bar{E}_{\mathrm{p}i}(\mathrm{NN}, j_\pi j_\pi{}')}{\Delta E_{\mathrm{p}i}}, \tag{25}$$

$$\bar{\varepsilon}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}') = \frac{\bar{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')}{\Delta E_{\mathrm{p}i}}, \tag{26}$$

where the averaged energies $\bar{E}_{\mathrm{p}i}(\mathrm{NN}, j_\pi j_\pi{}')$ and $\bar{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')$ are estimated, assuming the energy denominator $\Delta E_{\mathrm{p}i}$ to be constant. the factor $\tilde{\varepsilon}_{\mathrm{p}i}$ and the average energy $\tilde{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')$ are defined in the same way:

$$\tilde{\varepsilon}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}') = \frac{\tilde{E}_{\mathrm{p}i}(\mathrm{N}\Lambda, j_\pi j_\pi{}')}{\Delta E_{\mathrm{p}i}}. \tag{27}$$

In the numerical calculations we use free charges of nucleons and a Λ-hyperon ($e_\mathrm{p} = 1$, $e_\mathrm{n} = e_\Lambda = 0$).

From tables 3 and 4, we see that the number of correction terms $\delta_{\mathrm{N}\Lambda}$ is much fewer than that of correction terms

Table 3. The first-order corrections to quadrupole moments of medium-weight and heavy pΛ hypernuclei [$A = 50$ systems] (in efm^2). The corrections $\delta_{\mathrm{N}i}(\mathrm{NN})$ and $\delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$ are given in units of $1/\nu_{\mathrm{N}}$ and $1/\nu_{\Lambda}$, respectively. See text for details.

Hypernuclei	J^π	$\delta_{\mathrm{N}i}(\mathrm{NN})$	$\delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$
${}^{50}_{\Lambda}\mathrm{Sc}$	3^-	$-3.659\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 1f_{7/2}1p_{3/2})$	$-0.659\ \gamma_{\mathrm{N}2}(1f_{5/2})$
		$-2.250\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	$+2.244\ \gamma_{\mathrm{N}2}(2f_{7/2})$
		$-0.636\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	$+0.311\ \gamma_{\mathrm{N}2}(2f_{5/2})$
		$-0.734\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.367\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.519\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+1.909\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.367\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$-0.561\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.424\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	
		$-2.173\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-1.775\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.581\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.474\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{3/2})$	
		$+0.474\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-1.086\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	
		$+0.474\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}2p_{3/2})$	
${}^{50}_{\Lambda}\mathrm{Sc}$	4^-	$-4.098\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 1f_{7/2}1p_{3/2})$	$-0.479\ \gamma_{\mathrm{N}2}(1h_{9/2})$
		$-2.520\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	$+2.514\ \gamma_{\mathrm{N}2}(2f_{7/2})$
		$-0.712\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	
		$-0.823\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.411\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.582\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+2.138\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.411\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$+0.628\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.475\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	
		$-2.434\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-1.988\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.650\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.531\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{3/2})$	
		$+0.531\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-1.217\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	
		$+0.531\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}2p_{3/2})$	

δ_{NN}. Therefore, hyperon-induced configuration mixing effects ($\sum \delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$) turn out to be one or two orders of magnitude smaller than nucleon-induced configuration mixing effects ($\sum \delta_{\mathrm{N}i}(\mathrm{NN})$), because N$\Lambda$ interactions are one-order of magnitude smaller than NN interactions. More detailed numerical calculations of these corrections will be the next-step problem.

The perturbation theory, which we use in this and previous papers [3,6,7], turns out to work well in hypernuclear moments analyses, because NΛ interactions are one-order of magnitude smaller than NN interactions. Thus, we may justify the perturbation method in hypernuclear moment studies.

4 Discussions

In this section, we give discussions on quadrupole moments of special hypernuclei, whose effective values are available. We also discuss effective charges of nucleons and hyperons in hypernuclei and the diagram of hypernuclear quadrupole moments.

4.1 Quadrupole moments of ${}^{38}_{\Lambda}\mathrm{Cl}$

The hypernucleus ${}^{38}_{\Lambda}\mathrm{Cl}$ is not heavy, but we simply assume the doubled-closed core ${}^{36}_{16}\mathrm{S}_{20}$, which is the same type $N \neq Z$ core as ${}^{48}_{20}\mathrm{Ca}_{28}$ and ${}^{208}_{82}\mathrm{Pb}_{126}$ used in this paper. Thus, we studied quadrupole moments of ${}^{38}_{\Lambda}\mathrm{Cl}$ in this

Table 4. The first-order corrections to quadrupole moments of medium-weight and heavy nΛ hypernuclei [$A = 50$ systems] (in efm^2). The corrections $\delta_{\mathrm{N}i}(\mathrm{NN})$ and $\delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$ are given in units of $1/\nu_{\mathrm{N}}$ and $1/\nu_{\Lambda}$, respectively. See text for details.

Hypernuclei	J^π	$\delta_{\mathrm{N}i}(\mathrm{NN})$	$\delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$
${}^{50}_{\Lambda}\mathrm{Ca}$	1^-	$+1.897\ \bar{\varepsilon}_{\mathrm{p}1}(\mathrm{NN}, 1p_{3/2}1f_{7/2})$	$-0.422\ \tilde{\varepsilon}_{\mathrm{p}2}(\mathrm{N}\Lambda, 2p_{3/2}1p_{3/2})$
		$+0.517\ \bar{\varepsilon}_{\mathrm{p}2}(\mathrm{NN}, 2p_{3/2}1p_{3/2})$	$+0.422\ \tilde{\varepsilon}_{\mathrm{p}2}(\mathrm{N}\Lambda, 2p_{3/2}1p_{1/2})$
		$+0.517\ \bar{\varepsilon}_{\mathrm{p}2}(\mathrm{NN}, 2p_{3/2}1p_{1/2})$	$+0.516\ \tilde{\varepsilon}_{\mathrm{p}3}(\mathrm{N}\Lambda, 2p_{3/2}1p_{3/2})$
		$+0.632\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 2p_{3/2}1p_{3/2})$	$-0.516\ \tilde{\varepsilon}_{\mathrm{p}3}(\mathrm{N}\Lambda, 2p_{3/2}1p_{1/2})$
		$+0.632\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 2p_{3/2}1p_{1/2})$	
		$-1.227\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	
		$-0.347\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	
		$-0.400\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.200\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.283\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+1.041\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.200\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$+0.306\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.231\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	
		$-1.185\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-0.968\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.316\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.258\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-0.592\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	
		$-1.500\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	
		$-0.424\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	
		$-0.489\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.244\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.346\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+1.272\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.244\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$+0.374\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.282\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	
		$-1.449\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-1.183\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.387\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.316\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-0.724\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	
${}^{50}_{\Lambda}\mathrm{Ca}$	2^-	$+3.794\ \bar{\varepsilon}_{\mathrm{p}1}(\mathrm{NN}, 1p_{3/2}1f_{7/2})$	$+0.553\ \tilde{\varepsilon}_{\mathrm{p}2}(\mathrm{N}\Lambda, 2p_{3/2}1p_{3/2})$
		$+1.034\ \bar{\varepsilon}_{\mathrm{p}2}(\mathrm{NN}, 2p_{3/2}1p_{3/2})$	$-0.553\ \tilde{\varepsilon}_{\mathrm{p}2}(\mathrm{N}\Lambda, 2p_{3/2}1p_{1/2})$
		$+1.034\ \bar{\varepsilon}_{\mathrm{p}2}(\mathrm{NN}, 2p_{3/2}1p_{1/2})$	$-0.676\ \tilde{\varepsilon}_{\mathrm{p}3}(\mathrm{N}\Lambda, 2p_{3/2}1p_{3/2})$
		$+1.264\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 2p_{3/2}1p_{3/2})$	$+0.676\ \tilde{\varepsilon}_{\mathrm{p}3}(\mathrm{N}\Lambda, 2p_{3/2}1p_{1/2})$
		$+1.264\ \bar{\varepsilon}_{\mathrm{p}3}(\mathrm{NN}, 2p_{3/2}1p_{1/2})$	
		$-2.454\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	
		$-0.694\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	
		$-0.801\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.400\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.566\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+2.082\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.400\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$+0.612\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.462\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	

Table 4. continued

Hypernuclei	J^π	$\delta_{\mathrm{N}i}(\mathrm{NN})$	$\delta_{\mathrm{N}i}(\mathrm{N}\Lambda)$
		$-2.371\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-1.936\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.633\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.517\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-1.185\ \bar{\varepsilon}_{\mathrm{p}4}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	
		$-3.000\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{9/2})$	
		$-0.848\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}1g_{7/2})$	
		$-0.979\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{5/2})$	
		$-0.489\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}2d_{3/2})$	
		$-0.692\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{5/2}3s_{1/2})$	
		$+2.545\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}1g_{7/2})$	
		$-0.489\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{5/2})$	
		$+0.748\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}2d_{3/2})$	
		$-0.565\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1d_{3/2}3s_{1/2})$	
		$-2.898\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{5/2})$	
		$-2.366\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 2s_{1/2}2d_{3/2})$	
		$+0.774\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}1f_{5/2})$	
		$+0.632\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{3/2}2p_{1/2})$	
		$-1.499\ \bar{\varepsilon}_{\mathrm{p}5}(\mathrm{NN}, 1p_{1/2}1f_{5/2})$	

paper. The magnetic moment and quadrupole moment of ^{37}Cl were investigated by Noya *et al.* [5], and the experimental data [10] were nicely reproduced by the simple shell model. We may justify the simple shell model configuration $[1d_{3/2}(\mathrm{p}) \times 1s_{1/2}(\Lambda)]$ for the ground state of $^{38}_{\Lambda}$Cl. The effective quadrupole moments of 1^+ and 2^+ states of $^{38}_{\Lambda}$Cl are calculated to be $-3.246\,(\mathrm{efm}^2)$ and $-6.493\,(\mathrm{efm}^2)$, respectively.

4.2 Quadrupole moments of $^{210}_{\Lambda}$Bi

The hypernucleus $^{210}_{\Lambda}$Bi is a typical example of heavy pΛ systems, whose effective moments are available. The magnetic moment and quadrupole moment of ^{209}Bi ($9/2^-$, g.s.) were extensively studied by Noya *et al.* [5] with the use of the shell model. The magnetic moment was shown to be not satisfactorily reproduced by the configuration mixing theory. On the other hand, the quadrupole moment was shown to be nicely reproduced by the standard shell model calculation. By using the experimental data of ^{209}Bi ($9/2^-$, $-46.0\,\mathrm{efm}^2$) [10], quadrupole moments of 4^- and 5^- states of $^{210}_{\Lambda}$Bi are predicted to be $-42.9\,(\mathrm{efm}^2)$ and $-46.0\,(\mathrm{efm}^2)$, respectively. These values are several times enhanced in comparison with the single-particle values of the shell model (table 1).

4.3 Quadrupole moments of $^{62}_{\Lambda}$Ni

As is well known, Ni-isotopes have complex energy spectra, and valence neutrons have large effective charges [10]. For introducing effective quadrupole moments of heavier nΛ hypernuclei, we simply assumed the double-closed core $^{60}_{28}\mathrm{Ni}_{32}$ and calculated quadrupole moments of $^{62}_{\Lambda}$Ni in this paper. The quadrupole moment of the $5/2^-$ state ($0.067\,\mathrm{MeV}$) of ^{61}Ni was observed to be $-8.0\,(\mathrm{efm}^2)$ [10]. The effective quadrupole moments of the 2^- and 3^- state of $^{62}_{\Lambda}$Ni are calculated to be $-6.4\,(\mathrm{efm}^2)$ and $-8.0\,(\mathrm{efm}^2)$, respectively, which are compared to the single-particle value $0.0\,(\mathrm{efm}^2)$ (table 2).

4.4 Quadrupole moments of $^{74}_{\Lambda}$Ge

Ge-isotopes also have complex energy spectra [10]. As for the previous example ($^{62}_{\Lambda}$Ni), for the sake of introducing effective quadrupole moments of heavier nΛ hypernuclei, we simply assumed the double-closed core $^{72}_{32}\mathrm{Ge}_{40}$, and calculated quadrupole moments of $^{74}_{\Lambda}$Ge in this paper. The magnetic moment and quadrupole moment of ^{73}Ge were suited by Noya *et al.* [5], and the experimental data [10] were well reproduced by the simple shell model calculation.

The quadrupole moment of the $9/2^+$ state (g.s.) of ^{73}Ge was observed to be $-17.3\,(\mathrm{efm}^2)$ [10]. The effective quadrupole moments of 4^+ and 5^+ states of $^{74}_{\Lambda}$Ge are calculated to be $-16.1\,(\mathrm{efm}^2)$ and $-17.3\,(\mathrm{efm}^2)$, respectively. The experimental data of these moments are much desired to test the hypernuclei shell model.

4.5 Effective charges of nucleons and hyperons in hypernuclei

As is well known, neutrons and protons have effective charges (δe_{n} and δe_{p}) in nuclei, which shows the collective motions of nuclei [4,5]. Generally speaking, neutron's effective charges are rather larger than proton's ($\delta e_{\mathrm{n}} \gtrsim \delta e_{\mathrm{p}}$). From the shell model point of view, this fact is ascribed to the strong property of neutron-proton (np) interactions. Indeed, the number of correction terms in ${}^{50}_{\Lambda}\mathrm{Ca}$ are about twice larger than that of correction terms in ${}^{50}_{\Lambda}\mathrm{Sc}$ (tables 3 and 4). As NN interactions are one-order of magnitude larger than NΛ interactions, we may expect larger effective charges of nucleons and smaller effective charges of hyperons in Λ-hypernuclei; $\delta e_{\mathrm{n}} \gtrsim \delta e_{\mathrm{p}} \gtrsim \delta e_{\Lambda}$.

4.6 The quadrupole moment diagram of hypernuclei

The nuclear quadrupole moment diagram, where the ratios ($\mathcal{Q}_{\mathrm{exp}}/|\mathcal{Q}_{\mathrm{sp}}|$) are plotted against the mass number (A), is frequently used to show the deformation and the collective motion of nuclei [4,5]. In the same way, we may expect the hypernuclear quadrupole moment diagram, where the values ($\mathcal{Q}_{\mathrm{exp}}/|\mathcal{Q}_{\mathrm{sp}}|$) are plotted against the mass number (A). This diagram will lead us to investigate the deformation and the collective motion of hypernuclei. We desire the experimental observation and the diagram of hypernuclear quadrupole moments.

5 Summary

Finally, we summarize our work as follows. In this paper, we studied the quadrupole moments of medium-weight and heavy NΛ hypernuclei within the framework of the shell model. The quadrupole moments of these hypernuclei ($\mathcal{Q}$) are determined mainly by those of nuclei ($\mathcal{Q}_{\mathrm{N}}$), as far as the effective charge of the Λ-hyperon (e_{Λ}) is small. We also estimated the configuration mixing effects on these hypernuclear quadrupole moments by using the perturbation theory. The hyperon-induced configuration mixing effects presented in tables 3 and 4 turn out to be one or two orders of magnitude smaller than the nucleon-induced configuration mixing effects, because nucleon-hyperon (NΛ) interactions are one-order of magnitude smaller than nucleon-nucleon (NN) interactions. Therefore, the effective values of medium-weight and heavy NΛ hypernuclear quadrupole moments listed in tables 1 and 2 are expected to be very close to experimental moments. The experimental observation of hypernuclear quadrupole moments is much desired to test hypernuclear models and theories.

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Appendix A.

Using the basic states in the text ($\Psi_0, \Psi_{\mathrm{N1}}, \Psi_{\mathrm{N2}}, \Psi_{\mathrm{N3}}, \Psi_{\mathrm{N4}}$, and Ψ_{N5}), we obtained the correction formulae of configuration mixing effects on static moments of NΛ systems. In this appendix, we present the correction formulae, which we used in the numerical calculations of quadrupole moments. The symbol $f_{\mathrm{p}}^{(k)}$ denotes the one-body operator of rank k, such as the quadrupole moment $\mathcal{Q}_{\mathrm{p}}^{(k)} (k=2)$ and the symbol $\tilde{\delta}_{\mathrm{p}i}$ shows the reduced matrix element of the correction;

$$\delta_{\mathrm{p}i} = \begin{pmatrix} J & k & J \\ -J & 0 & J \end{pmatrix} \tilde{\delta}_{\mathrm{p}i} \quad (i = 1,2,3,4,5).$$

Appendix A.1. The pΛ systems with $\mathbf{j_p} = \mathbf{l_p} \pm 1/2$ orbit

1) Correction induced by Ψ_{N1} (NΛ interaction).

$$\begin{aligned}\tilde{\delta}_{\mathrm{p1}}(\mathrm{N}\Lambda) &= 2\sum_{j_{\mathrm{p}}'} \langle \Psi_0 \| f_{\mathrm{p}}^{(k)} \| \Psi_{\mathrm{N1}}(j_{\mathrm{p}}') \rangle_J \, \gamma_{\mathrm{N1}}(j_{\mathrm{p}}'), \\ &= 2\sum_{j_{\mathrm{p}}'} (-)^{J+k+j_{\mathrm{p}}+j_{\Lambda}} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\mathrm{p}} & j_{\mathrm{p}}' & j_{\Lambda} \end{Bmatrix} \\ &\quad \times \langle j_{\mathrm{p}} \| f_{\mathrm{p}}^{(k)} \| j_{\mathrm{p}}' \rangle \gamma_{\mathrm{N1}}(j_{\mathrm{p}}'). \end{aligned} \qquad \text{(A.1)}$$

2) Correction induced by Ψ_{N2} (NΛ interaction).

$$\begin{aligned}\tilde{\delta}_{\mathrm{p2}}(\mathrm{N}\Lambda) &= 2\sum_{j_{\mathrm{p}}'} \langle \Psi_0 \| f_{\mathrm{p}}^{(k)} \| \Psi_{\mathrm{N2}}(j_{\mathrm{p}}') \rangle_J \, \gamma_{\mathrm{N2}}(j_{\mathrm{p}}'), \\ &= 2a^2 \sum_{j_{\mathrm{p}}'} (-)^{J+k+j_{\mathrm{p}}+j_{\Lambda}+1} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\mathrm{p}} & j_{\mathrm{p}}' & j_{\Lambda} \end{Bmatrix} \\ &\quad \times \langle j_{\mathrm{p}} \| f_{\mathrm{p}}^{(k)} \| j_{\mathrm{p}}' \rangle \gamma_{\mathrm{N2}}(j_{\mathrm{p}}'). \end{aligned} \qquad \text{(A.2)}$$

3) Correction induced by Ψ_{N3} (NN interaction).

$$\begin{aligned}\tilde{\delta}_{\mathrm{p3}}(\mathrm{NN}) &= -\frac{2}{\Delta E_{\mathrm{N3}}} \sum_{j_{\mathrm{p}}'} \sum_{J_0 J_{\mathrm{N}}} \langle \Psi_0 \| f_{\mathrm{p}}^{(k)} \| \Psi_{\mathrm{p3}} \rangle_J \\ &\quad \times \langle \Psi_{\mathrm{p3}} | V_{\mathrm{NN}} | \Psi_0 \rangle_J, \\ &= 2\sum_{j_{\mathrm{p}}'} (-)^{J+k+j_{\mathrm{p}}'+j_{\Lambda}+1} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\mathrm{p}} & j_{\mathrm{p}} & j_{\Lambda} \end{Bmatrix} \\ &\quad \times \langle j_{\mathrm{p}} \| f_{\mathrm{p}}^{(k)} \| j_{\mathrm{p}}' \rangle \frac{\bar{E}_{\mathrm{p3}}(\mathrm{NN}, j_{\mathrm{p}} j_{\mathrm{p}}')}{\Delta E_{\mathrm{N3}}}, \end{aligned} \qquad \text{(A.3)}$$

where

$$\begin{aligned}\bar{E}_{\mathrm{p3}}(\mathrm{NN}, j_{\mathrm{p}} j_{\mathrm{p}}') &= \sum_{J_0} (2J_0+1) \begin{Bmatrix} j_{\mathrm{p}} & j_{\mathrm{p}} & J_0 \\ j_{\mathrm{p}}' & j_{\mathrm{p}} & k \end{Bmatrix} \\ &\quad \times \langle j_{\mathrm{p}}^2 | V_{\mathrm{NN}} | j_{\mathrm{p}}' j_{\mathrm{p}} \rangle_{J_0} \; (J_0 = 0, \mathrm{even}). \end{aligned} \qquad \text{(A.4)}$$

4) Correction induced by $\Psi_{\rm N3}$ (NΛ interaction).

$$\tilde{\delta}_{\rm p3}({\rm NN}) = -\frac{2}{\Delta E_{\rm N3}} \sum_{j_{\rm p}'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p3} \rangle_J \times \langle \Psi_{\rm p3} | V_{{\rm N}\Lambda} | \Psi_0 \rangle_J \times \frac{1+(-)^{J_0}}{2}, = \sum_{j_{\rm p}'} (-)^{J+k+j_{\rm p}+j_{\rm p}'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_{\rm p} \end{Bmatrix} \times \langle j_{\rm p} \| f_{\rm p}^{(k)} \| j_{\rm p}' \rangle \frac{\bar{E}_{\rm p3}({\rm N}\Lambda, j_{\rm p} j_{\rm p}')}{\Delta E_{\rm N3}} + \sum_{j_{\rm p}'} (-)^{J+k+j_{\rm p}+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\rm p} & j_{\rm p}' & j_\Lambda \end{Bmatrix} \times \langle j_{\rm p} \| f_{\rm p}^{(k)} \| j_{\rm p}' \rangle \frac{\langle j_{\rm p} j_\Lambda | V_{{\rm N}\Lambda} | j_{\rm p}' j_\Lambda \rangle_J}{\Delta E_{\rm N3}}, \quad \text{(A.5)}$$

where

$$\bar{E}_{\rm p3}({\rm N}\Lambda, j_{\rm p} j_{\rm p}') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_{\rm p} & j_\Lambda & J' \\ j_\Lambda & j_{\rm p}' & k \end{Bmatrix} \times \langle j_{\rm p} j_\Lambda | V_{{\rm N}\Lambda} | j_{\rm p}' j_\Lambda \rangle_{J'}. \quad \text{(A.6)}$$

5) Correction induced by $\Psi_{\rm N4}$ (NN interaction).

$$\tilde{\delta}_{\rm p4}({\rm NN}) = -\frac{2}{\Delta E_{\rm N4}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p4} \rangle_J \times \langle \Psi_{\rm p4} | V_{\rm NN} | \Psi_0 \rangle_J, = 2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_\pi'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\rm p} & j_{\rm p} & j_\Lambda \end{Bmatrix} \times \langle j_\pi \| f_{\rm p}^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{\rm p4}({\rm NN}, j_\pi j_\pi')}{\Delta E_{\rm N4}}, \quad \text{(A.7)}$$

where

$$\bar{E}_{\rm p4}({\rm NN}, j_\pi j_\pi') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi' & j_{\rm p} & J' \\ j_{\rm p} & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_{\rm p} | V_{\rm NN} | j_\pi j_{\rm p} \rangle_{J'}. \quad \text{(A.8)}$$

6) Correction induced by $\Psi_{\rm N4}$ (NΛ interaction).

$$\tilde{\delta}_{\rm p4}({\rm N}\Lambda) = -\frac{2}{\Delta E_{\rm N4}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p4} \rangle_J \times \langle \Psi_{\rm p4} | V_{{\rm N}\Lambda} | \Psi_0 \rangle_J, = 2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_{\rm p}+j_\pi'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_{\rm p} \end{Bmatrix} \times \langle j_\pi \| f_{\rm p}^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{\rm p4}({\rm N}\Lambda, j_\pi j_\pi')}{\Delta E_{\rm N4}}, \quad \text{(A.9)}$$

where

$$\bar{E}_{\rm p4}({\rm N}\Lambda, j_\pi j_\pi') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi' & j_\Lambda & J' \\ j_\Lambda & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_\Lambda | V_{{\rm N}\Lambda} | j_\pi j_\Lambda \rangle_{J'}. \quad \text{(A.10)}$$

7) Correction induced by $\Psi_{\rm N5}$ (NN interaction).

$$\tilde{\delta}_{\rm p5}({\rm NN}) = -\frac{2}{\Delta E_{\rm N5}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p5} \rangle_J \times \langle \Psi_{\rm p5} | V_{\rm NN} | \Psi_0 \rangle_J, = 2\alpha^2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_\pi'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\rm p} & j_{\rm p} & j_\Lambda \end{Bmatrix} \times \langle j_\pi \| f_{\rm p}^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{\rm p5}({\rm NN}, j_\pi j_\pi')}{\Delta E_{\rm N5}}, \quad \text{(A.11)}$$

where

$$\bar{E}_{\rm p5}({\rm NN}, j_\pi j_\pi') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi' & j_{\rm p} & J' \\ j_{\rm p} & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_{\rm p} | V_{\rm NN} | j_\pi j_{\rm p} \rangle_{J'}. \quad \text{(A.12)}$$

8) Correction induced by $\Psi_{\rm N5}$ (NΛ interaction).

$$\tilde{\delta}_{\rm p5}({\rm N}\Lambda) = -\frac{2}{\Delta E_{\rm N5}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p5} \rangle_J \times \langle \Psi_{\rm p5} | V_{{\rm N}\Lambda} | \Psi_0 \rangle_J, = 2\alpha^2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_{\rm p}+j_\pi'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_{\rm p} \end{Bmatrix} \times \langle j_\pi \| f_{\rm p}^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{\rm p5}({\rm N}\Lambda, j_\pi j_\pi')}{\Delta E_{\rm N5}}, \quad \text{(A.13)}$$

where

$$\bar{E}_{\rm p5}({\rm N}\Lambda, j_\pi j_\pi') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi' & j_\Lambda & J' \\ j_\Lambda & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_\Lambda | V_{{\rm N}\Lambda} | j_\pi j_\Lambda \rangle_{J'}. \quad \text{(A.14)}$$

Appendix A.2. The nΛ systems with $\mathbf{j_n = l_n \pm 1/2}$ orbit

1) Correction induced by $\Psi_{\rm N1}$ (NN interaction).

$$\tilde{\delta}_{\rm p1}({\rm NN}) = -\frac{2}{\Delta E_{\rm N1}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_{\rm N}} \langle \Psi_0 \| f_{\rm p}^{(k)} \| \Psi_{\rm p1} \rangle_J \times \langle \Psi_{\rm p1} | V_{\rm NN} | \Psi_0 \rangle_J, = 2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_\pi'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_{\rm n} & j_{\rm n} & j_\Lambda \end{Bmatrix} \times \langle j_\pi \| f_{\rm p}^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{\rm p1}({\rm NN}, j_\pi j_\pi')}{\Delta E_{\rm N1}}, \quad \text{(A.15)}$$

where

$$\bar{E}_{\rm p1}({\rm NN}, j_\pi j_\pi') = \sum_{J_0} (-)^{J_0} (2J_0+1) \begin{Bmatrix} j_\pi' & j_{\rm n} & J_0 \\ j_{\rm n} & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_{\rm n} | V_{\rm NN} | j_\pi j_{\rm n} \rangle_{J_0}. \quad \text{(A.16)}$$

2) Correction induced by Ψ_{N1} (NΛ interaction).

$$\tilde{\delta}_{p1}(N\Lambda) = -\frac{2}{\Delta E_{N1}} \sum_{j_\pi j_\pi'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p1} \rangle_J \times \langle \Psi_{p1} | V_{N\Lambda} | \Psi_0 \rangle_J, = 2 \sum_{j_\pi j_\pi'} (-)^{J+k+j_n+j_\pi'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_n \end{Bmatrix} \times \langle j_\pi \| f_p^{(k)} \| j_\pi' \rangle \frac{\bar{E}_{p1}(N\Lambda, j_\pi j_\pi')}{\Delta E_{N1}}, \quad (A.17)$$

where

$$\bar{E}_{p1}(N\Lambda, j_\pi j_\pi') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi' & j_\Lambda & J' \\ J_\Lambda & j_\pi & k \end{Bmatrix} \times \langle j_\pi' j_\Lambda | V_{N\Lambda} | j_\pi j_\Lambda \rangle_{J'}. \quad (A.18)$$

3) Correction induced by Ψ_{N2} (NN interaction).

$$\tilde{\delta}_{p2}(NN) = -\frac{2}{\Delta E_{N2}} \sum_{j_p'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p2} \rangle_J \times \langle \Psi_{p2} | V_{NN} | \Psi_0 \rangle_J, = 2\tilde{\alpha}^2 \sum_{j_p'} (-)^{J+k+j_p'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_n & j_n & j_\Lambda \end{Bmatrix} \times \langle j_p \| f_p^{(k)} \| j_p' \rangle \frac{\bar{E}_{p2}(NN, j_p j_p')}{\Delta E_{N2}}, \quad (A.19)$$

where

$$\bar{E}_{p2}(NN, j_p\, j_p') = \sum_{J_0} (-)^{J_0} (2J_0+1) \begin{Bmatrix} j_p' & j_n & J_0 \\ j_n & j_p & k \end{Bmatrix} \times \langle j_p j_n | V_{NN} | j_p' j_n \rangle_{J_0} \quad (J_0 = 0, \text{even}). \quad (A.20)$$

4) Correction induced by Ψ_{N2} (NΛ interaction).

$$\tilde{\delta}_{p2}(N\Lambda) = -\frac{2}{\Delta E_{N2}} \sum_{j_p'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p2} \rangle_J \times \langle \Psi_{p2} | V_{N\Lambda} | \Psi_0 \rangle_J \frac{1+(-)^{J_0}}{2}, = \tilde{\alpha}^2 \sum_{j_p'} (-)^{J+k+j_p'+j_n} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_n \end{Bmatrix} \times \langle j_p \| f_p^{(k)} \| j_p' \rangle \frac{\bar{E}_{p2}(N\Lambda, j_p j_p')}{\Delta E_{N2}} + \tilde{\alpha}^2 \sum_{j_p'} (-)^{J+k+j_p+j_\Lambda} \langle j_p \| f_p^{(k)} \| j_p' \rangle \times \frac{\tilde{E}_{p2}(N\Lambda, j_p j_p')}{\Delta E_{N2}}, \quad (A.21)$$

where

$$\bar{E}_{p2}(N\Lambda, j_p j_p') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_p & j_\Lambda & J' \\ j_\Lambda & j_p' & k \end{Bmatrix} \times \langle j_p j_\Lambda | V_{N\Lambda} | j_p' j_\Lambda \rangle_{J'}, \quad (A.22)$$

and

$$\tilde{E}_{p2}(N\Lambda, j_p j_p') = \sum_{J'} (2J'+1) \begin{Bmatrix} j_p' & j_\Lambda & J' \\ J & k & j_n \end{Bmatrix} \times \langle j_p j_\Lambda | V_{N\Lambda} | j_p' j_\Lambda \rangle_{J'}. \quad (A.23)$$

5) Correction induced by Ψ_{N3} (NN interaction).

$$\tilde{\delta}_{p3}(NN) = -\frac{2}{\Delta E_{N3}} \sum_{j_p'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p3} \rangle_J \times \langle \Psi_{p3} | V_{NN} | \Psi_0 \rangle_J, = 2 \sum_{j_p'} (-)^{J+k+j_p'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_n & j_n & j_\Lambda \end{Bmatrix} \times \langle j_p \| f_p^{(k)} \| j_p' \rangle \frac{\bar{E}_{p3}(NN, j_p j_p')}{\Delta E_{N3}}, \quad (A.24)$$

where

$$\bar{E}_{p3}(NN, j_p\, j_p') = \sum_{J_0} (-)^{J_0} (2J_0+1) \begin{Bmatrix} j_p' & j_n & J_0 \\ j_n & j_p & k \end{Bmatrix} \times \langle j_p j_n | V_{NN} | j_p' j_n \rangle_{J_0} \quad (J_0 = \text{odd}). \quad (A.25)$$

6) Correction induced by Ψ_{N3} (NΛ interaction).

$$\tilde{\delta}_{p3}(N\Lambda) = -\frac{2}{\Delta E_{N3}} \sum_{j_p'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p3} \rangle_J \times \langle \Psi_{N3} | V_{N\Lambda} | \Psi_0 \rangle_J \times \frac{1-(-)^{J_0}}{2}, = \sum_{j_p'} (-)^{J+k+j_p'+j_n} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_n \end{Bmatrix} \times \langle j_p \| f_p^{(k)} \| j_p' \rangle \frac{\bar{E}_{p3}(N\Lambda, j_p j_p')}{\Delta E_{N3}} - \sum_{j_p'} (-)^{J+k+j_p+j_\Lambda} \langle j_p \| f_p^{(k)} \| j_p' \rangle \times \frac{\tilde{E}_{p3}(N\Lambda, j_p j_p')}{\Delta E_{N3}}, \quad (A.26)$$

where

$$\bar{E}_{p3}(N\Lambda, j_p j_p') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_p & j_\Lambda & J' \\ j_\Lambda & j_p' & k \end{Bmatrix} \times \langle j_p j_\Lambda | V_{N\Lambda} | j_p' j_\Lambda \rangle_{J'}, \quad (A.27)$$

and

$$\tilde{E}_{p3}(N\Lambda, j_p j_p') = \sum_{J'} (2J'+1) \begin{Bmatrix} j_p' & j_\Lambda & J' \\ J & k & j_n \end{Bmatrix} \times \langle j_p j_\Lambda | V_{N\Lambda} | j_p' j_\Lambda \rangle_{J'}. \quad (A.28)$$

7) Correction induced by Ψ_{N4} (NN interaction).

$$\tilde{\delta}_{p4}(NN) = -\frac{2}{\Delta E_{N4}} \sum_{j_\pi j_\pi{}'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p4} \rangle_J \times \langle \Psi_{p4} | V_{NN} | \Psi_0 \rangle_J, = 2\alpha^2 \sum_{j_\pi j_\pi{}'} (-)^{J+k+j_\pi{}'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_n & j_n & j_\Lambda \end{Bmatrix} \times \langle j_\pi \| f_p^{(k)} \| j_\pi{}' \rangle \frac{\bar{E}_{p4}(NN, j_\pi j_\pi{}')}{\Delta E_{N4}}, \quad (A.29)$$

where

$$\bar{E}_{p4}(NN, j_\pi j_\pi{}') = \sum_{J_0} (-)^{J_0} (2J_0+1) \begin{Bmatrix} j_\pi{}' & j_n & J_0 \\ j_n & j_\pi & k \end{Bmatrix} \times \langle j_\pi{}' j_n | V_{NN} | j_\pi j_n \rangle_{J_0, T=1}. \quad (A.30)$$

8) Correction induced by Ψ_{N4} (NΔ interaction).

$$\tilde{\delta}_{p4}(N\Lambda) = -\frac{2}{\Delta E_{N4}} \sum_{j_\pi j_\pi{}'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p4} \rangle_J \times \langle \Psi_{p4} | V_{N\Lambda} | \Psi_0 \rangle_J, = 2\alpha^2 \sum_{j_\pi j_\pi{}'} (-)^{J+k+j_n+j_\pi{}'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_n \end{Bmatrix} \times \langle j_\pi \| f_p^{(k)} \| j_\pi{}' \rangle \frac{\bar{E}_{p3}(N\Lambda, j_\pi j_\pi{}')}{\Delta E_{N4}}, \quad (A.31)$$

where

$$\bar{E}_{p4}(N\Lambda, j_\pi j_\pi{}') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi{}' & j_\Lambda & J' \\ j_\Lambda & j_\pi & k \end{Bmatrix} \times \langle j_\pi{}' j_\Lambda | V_{N\Lambda} | j_\pi j_\Lambda \rangle_{J'}. \quad (A.32)$$

9) Correction induced by Ψ_{N5} (NN interaction).

$$\tilde{\delta}_{p5}(NN) = -\frac{2}{\Delta E_{N5}} \sum_{j_\pi j_\pi{}'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p5} \rangle_J \times \langle \Psi_{p5} | V_{NN} | \Psi_0 \rangle_J, = \sum_{j_\pi j_\pi{}'} (-)^{J+k+j_\pi{}'+j_\Lambda} (2J+1) \begin{Bmatrix} J & J & k \\ j_n & j_n & j_\Lambda \end{Bmatrix} \times \langle j_\pi \| f_p^{(k)} \| j_\pi{}' \rangle \frac{\bar{E}_{p5}(NN, j_\pi j_\pi{}')}{\Delta E_{N5}}, \quad (A.33)$$

where

$$\bar{E}_{p5}(NN, j_\pi j_\pi{}') = \sum_{J_0} (-)^{J_0} (2J_0+1) \begin{Bmatrix} j_\pi{}' & j_n & J_0 \\ j_n & j_\pi & k \end{Bmatrix} \times \langle j_\pi{}' j_n | V_{NN} | j_\pi j_n \rangle_{J_0, T_0=0}. \quad (A.34)$$

10) Correction induced by Ψ_{N5} (NΛ interaction).

$$\tilde{\delta}_{p5}(N\Lambda) = -\frac{2}{\Delta E_{N5}} \sum_{j_\pi j_\pi{}'} \sum_{J_0 J_N} \langle \Psi_0 \| f_p^{(k)} \| \Psi_{p5} \rangle_J \times \langle \Psi_{p5} | V_{N\Lambda} | \Psi_0 \rangle_J, = \sum_{j_\pi j_\pi{}'} (-)^{J+k+j_n+j_\pi{}'} (2J+1) \begin{Bmatrix} J & J & k \\ j_\Lambda & j_\Lambda & j_n \end{Bmatrix} \times \langle j_\pi \| f_p^{(k)} \| j_\pi{}' \rangle \frac{\bar{E}_{p5}(N\Lambda, j_\pi j_\pi{}')}{\Delta E_{N5}}, \quad (A.35)$$

where

$$\bar{E}_{p5}(N\Lambda, j_\pi j_\pi{}') = \sum_{J'} (-)^{J'} (2J'+1) \begin{Bmatrix} j_\pi{}' & j_\Lambda & J' \\ j_\Lambda & j_\pi & k \end{Bmatrix} \times \langle j_\pi{}' j_\Lambda | V_{N\Lambda} | j_\pi j_\Lambda \rangle_{J'}. \quad (A.36)$$

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